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Analysis of Cotton Fiber Maturity. III. A Study of Breaking Load Distribution of Single Fibers

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Synopsis

Breaking load frequency distributions have been obtained at different test gauge lengths for several raw cottons differing in fiber cell wall maturity. The curve-fitting procedure for the analysis of the shape of observed breaking load histograms is described. The procedure yields several important parameters characterizing the raw cotton, such as the mean, mode, skewness, standard deviation, and the C.V. of breaking load. It is found that for all raw cottons, the observed breaking load distributions have a positive skew and are of β type. The effect of fiber maturity and the test gauge length on the shape of observed distributions has been analyzed. The effect of weak links on the variability of breaking load in raw cotton is discussed.

INTRODUCTION

In the routine testing of textile fibers either as a means of quality control or to provide information on the effect of a treatment, it is frequently necessary to measure the tensile strengths of single fibers. A knowledge of the breaking strength is of vital importance in determining the strength of the yarns spun from them.[1] The rupture of single cotton fibers under load occurs preferentially at weak places along the fiber length.

During the process of fiber growth,[2] the average breaking load of single cotton fibers is known to increase with increasing fiber weight or cell wall thickness. A study of the breaking load frequency curves[3–7] may provide an insight into the phenomenon of fiber cell wall rupture as well as explain the causes of variability of fiber strength. There can be several factors which may cause irregularity and variability of the breaking load, such as the instrumental factors, test gauge length, intrinsic fiber quality, etc. For instance, it is known[4,5] that the asymmetry or skewness of the breaking load frequency curve for cotton arises from the superabundance of weak fibers, i.e., the fibers which are weak due to structural abnormalities and weak points (such as stress concentrations, thin places, or cracks in the fiber cell wall) which result from growth factors or from damage to the cell wall due to bacterial attack or during ginning. Furthermore, the breaking load frequency curve is known to become more regular[4] after the removal of such weak or damaged fibers.

It was Turner[3] who first obtained the breaking load frequency curves for cotton fibers, yarns, and fabrics. Clegg[4] determined the distributions as well as mean values of wall thickness and breaking load for a few cottons. However, in her work, the shapes of these distributions were not analyzed. No definite relationship was found between wall thickness and average breaking load, and it was concluded that finer cottons with thinner walls do not necessarily possess lower breaking strengths. Koshal and Turner[5] obtained the frequency curves for various fiber properties of cotton such as length, width, convolutions, strength, and rigidity. They found the breaking load for cotton to have a Pearson Type I distribution. They did not observe any change in the form of the breaking load distribution upon using whole fiber lengths instead of 1-cm lengths for testing or upon drastically reducing the total number of single fibers used in tests.

The present paper concerns itself with the objective of obtaining the breaking load distributions for several raw cottons and analyzing their form by curve fitting, which yields information regarding the asymmetry, variability, and average value of the breaking load. The effects of weak links in cotton on the breaking load has been analyzed by obtaining the distributions at different test gauge lengths. The effect of fiber maturity on the breaking load distribution in raw cotton has also been examined.

EXPERIMENTAL

Materials

Breaking load distributions have been studied for six cottons covering a wide range in fiber maturity. These cottons, which were the subject of an earlier investigation,[8] include five American varieties (USDA Code Nos. 888, 875, 805, 193, and 940) and Egyptian Karnak. Cotton 888 belongs to the Iquitos variety; it is very coarse, and microscopic studies of fibers have revealed that it has a very cylindrical shape and a very small lumen. Cotton 875, which has the highest percentage of mature fibers, is a breeder's sample. The rest of the American cottons belong to the upland variety.

Testing

The breaking loads for various raw cottons, preconditioned at 27°C and 65% R.H., were obtained using the Instron tensile tester. In each case, a random sample of 300 fibers was selected and individual fibers broken and the results classified into frequency arrays. The test gauge length was varied from 2 mm to 20 mm in order to study the weak link effect. The normal test length for single cotton fibers is 10 mm. The work of rupture was obtained from the area under the stress–strain curves, and the breaking tenacity was determined by dividing the average breaking load by the mean gravimetric fineness of the cotton sample.

Curve Fitting Procedure for Analyzing the Breaking Load Histograms

The procedure for analyzing the form of the breaking load frequency curves[9] is briefly summarized as follows:

(i) The frequencies f_i were determined from the observed breaking load results and plotted as a function of the midpoint of the (load) class interval. The number p of class intervals has been taken to be $\alpha + 1$, where α is the largest integer with $2^{\alpha-1} \leqslant n$ and $\sqrt{n} \leqslant p \leqslant \alpha$. This is between 9 and 17, and the number p has been made to remain the same to within rounding-off errors.

(ii) Various central moments of the observed frequency curve up to the fourth (μ_2 to μ_4) were computed. Sheppard's corrections[10] were applied to μ_2 and μ_4.

(iii) From the values of the moments, $\beta_1 = \mu_3^2/\mu_2^3$ and $\beta_2 = \mu_4/\mu_2^2$ were calculated.

(iv) From the values of β_1 and β_2, the *k factor* was determined:

$$k = \frac{\beta_1(\beta_2 + 3)^2}{4(2\beta_2 - 3\beta_1 - 6)(4\beta_2 - 3\beta_1)} \quad (1)$$

(v) The k criterion is known to provide information about the nature of the frequency distribution. In the present case, the analysis of frequency data of raw Egyptian Karnak using the k criterion has shown the breaking load distributions to be Karl Pearson's Type I, namely, the β type. The β type density function is given by[9]

$$y = Kx^{m-1}(a - x)^{n-1} \quad (2)$$

where y is the frequency of occurrence of any breaking load x, K is a normalizing constant, a is the total range of the frequency curve, and m and n are parameters governing the asymmetry of the form of the frequency curve.

(vi) The parameters m and n were estimated from the observed frequency data by the method of moments, and a was estimated by the chi-square (χ^2) method. The χ^2 minimum method for estimating the range a has been used in view of the sensitivity of the estimate obtained by the method of moments. The search method has been used to minimize chi-square for obtaining a. The frequency curve was computed in terms of m, n, and a. The frequencies, e_i, were obtained approximately for each value of a by calculating the value of the frequency curve at the midpoints of class intervals and multiplying by the length of the class interval. This had to be done in view of the impracticability of using either a better numerical procedure or using the tables of incomplete β functions, both of which are time consuming even while using a computer. The minimum value of chi-square;

$$\chi^2 = \sum \frac{(e_i - f_i)^2}{e_i} \quad (3)$$

has been used to test the goodness of fit. Three parameters, m, n, and a, have been estimated from the data, and the number of degrees of freedom in all the cases for χ^2 is therefore $(p - q - 1)$, where q = number of parameters; hence, the number of degrees of freedom is $(p - 4)$.

(vii) From the values of m, n, and a, it is possible to obtain the mode of the breaking load frequency curve:

$$\text{mode} = \frac{(m - 1)a}{(m + n - 2)} \quad (4)$$

skewness of the breaking load frequency curve was obtained from the formula

$$\text{skewness} = \frac{\text{mean} - \text{mode}}{\text{standard deviation}} \quad (5)$$

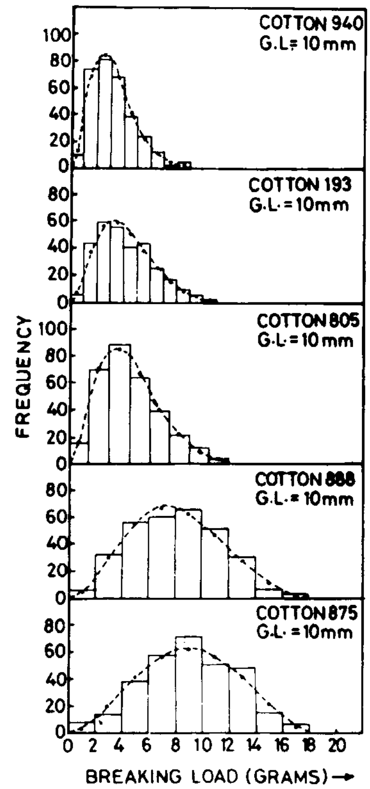

Fig. 1. Calculated fits (broken line) of β distributions to the observed breaking load histograms of various raw cotton of differing maturity (gauge length = 10 mm).

RESULTS

The β_1, β_2, and k factor for the observed breaking load frequency curves of raw Egyptian Karnak are listed in Table I. The values obtained for β_1 and β_2 have been referred to Tables 34.B and 34.C of the Biometrika Tables[11] and have been found to be smaller than the significance values at 5%. The data represent the frequency curves obtained at three different gauge lengths, namely, 2, 6, and 10 mm. It can be seen that the k factor is negative in all cases, implying that all the breaking load frequency curves can be fitted by a β distribution. Empirical

TABLE I
Constants Determining the Type of the Frequency Curve for Egyptian Karnak Cotton

Sample	Test gauge length, mm	β_1	β_2	k Factor	Type of breaking load frequency curve
	2	0.1692	3.0843	−0.3905	β
Raw cotton	6	0.0312	2.3273	−0.0167	β
	10	0.0552	2.3446	−0.0290	β

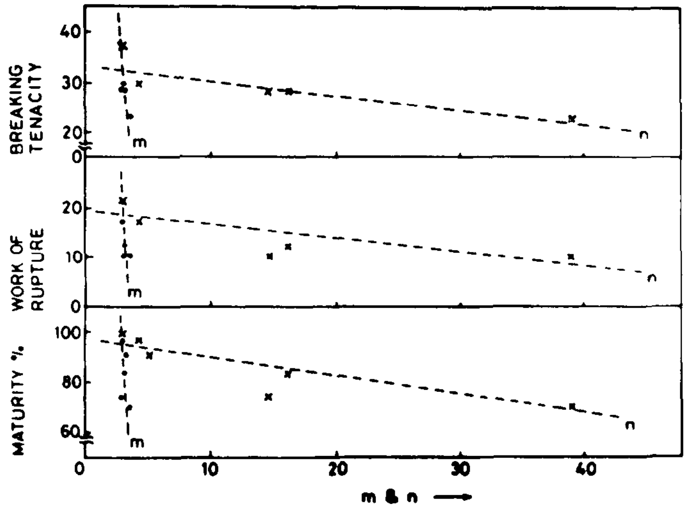

Fig. 2. Plot of frequency constants m and n vs. per cent mature fibers, work of rupture, and tenacity at break.

fits of the β distribution have, therefore, been attempted on frequency curves of all raw cottons, obtained using different gauge lengths.

The observed breaking load frequencies for various raw cottons are plotted in Figures 1 and 4 as histograms. The correctness of fit of the β distributions to these histograms is indicated by smooth curves. Observed and calculated frequencies of breaking load at 10-mm gauge length are listed in Table II and plotted in Figure 1 for five raw cottons of differing maturity. The frequency data at other gauge lengths have not been tabulated in this manner, but are illustrated graphically in Figure 4.

Figure 2 shows a plot of the frequency constants m and n against the per cent mature fibers, the average work of rupture, and the average breaking tenacity. It can be seen that while m is practically independent of any of these fiber

TABLE II
Observed and Calculated Frequency Distributions of Breaking Load for Raw Cottons[a]

Cotton 875			888			805			193			940		
x_i	f_i	e_i	x_i	f_i	e_i	x_i	f_i	e_i	x_i	f_i	e_i	x_i	f_i	e_i
1	8	3	1	5	5	0.75	16	16	0.5	5	8	0.5	10	12
3	14	21	3	30	30	2.25	70	71	1.5	43	38	1.5	74	68
5	38	41	5	57	55	3.75	88	86	2.5	59	57	2.5	82	86
7	57	56	7	59	66	5.25	64	67	3.5	56	59	3.5	68	67
9	71	62	9	64	62	6.75	39	41	4.5	40	51	4.5	38	42
11	50	57	11	50	46	8.25	21	21	5.5	43	37	5.5	24	22
13	48	41	13	30	27	9.75	12	9	6.5	25	25	6.5	12	10
15	14	21	15	6	11	11.25	4	3	7.5	17	15	7.5	2	4
17	6	4	17	3	2		—	—	8.5	9	8	8.5	4	2
19	—		19	—	—		—	—	9.5	5	4		—	—
									10.5	1	2		—	—

[a] Gauge length: 10 mm. x_i = Midpoint of class (g); f_i = observed frequency; e_i = calculated frequency.

TABLE III
Effect of Cotton Fiber Maturity on the Variability of Breaking Load[a]

Sample no.	Sample particulars	Per cent mature fibers p_M	Gravimetric fineness $\times 10^{-8}$, g/cm	Constants of calculated frequency curve			Goodness of chi-square x^2	Mean single fiber breaking load, g	Standard deviation, g	% Coefficient of variation of breaking load, g	Mode, g	Skewness
				m	n	Range a						
1	Cotton 875	98	236	2.9	2.9	18.1	15.8	9.0	3.5	38.6	9.0	0.0
2	Cotton 888	96	296	3.0	4.3	19.5	4.2	8.0	3.4	42.2	7.3	0.2
3	Cotton Egyptian Karnak	90	127	3.3	5.2	12.8	6.8	5.0	2.0	40.8	4.5	0.3
4	Cotton 805	83	207	3.1	16.2	28.3	1.4	4.5	2.2	49.7	3.4	0.5
5	Cotton 193	73	174	3.0	14.6	24.5	5.3	3.8	2.1	55.8	3.1	0.3
6	Cotton 940	69	169	3.5	39.0	38.5	6.2	3.0	1.6	52.4	2.4	0.4

[a] Gauge length: 10 mm.

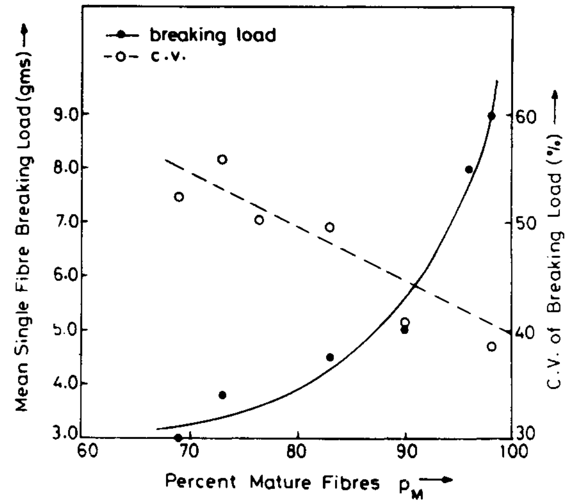

Fig. 3. Plot of C.V. and average breaking load vs. per cent mature fibers p_M.

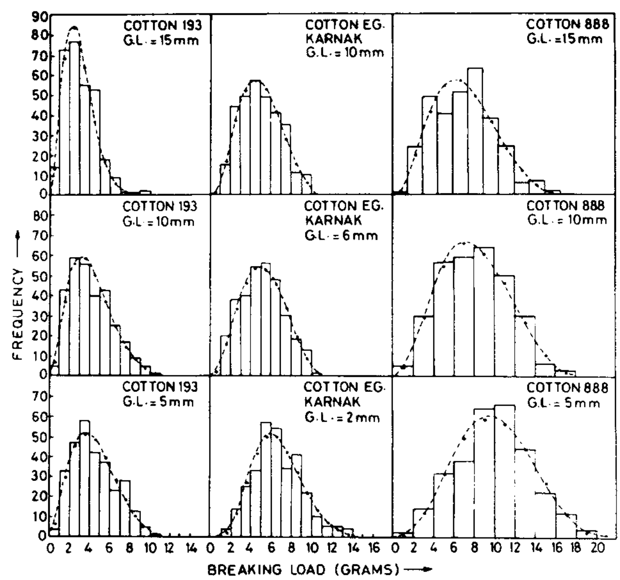

Fig. 4. Effect of test length on variability of breaking load in three raw cottons of different p_M.

characteristics, n bears a definite relationship to each of them. The mean breaking load and the C.V. of the breaking load (Table III) are plotted as a function of the per cent mature fibers in Figure 3. It can be seen from Figure 3 that with increasing fiber maturity, there is a steady increase in the average single-fiber breaking load and a decrease in its C.V.

The effect of fiber maturity on the variability of breaking load is illustrated in Figure 1 and listed in Table III, which shows various frequency constants of the fitted β distributions as well as χ^2 values obtained. With increasing maturity, the parameter n decreases and the range a of the breaking load tends to become narrower. It can be seen from Table III that all the breaking load frequency

TABLE IV
Effect of Gauge Length on the Variability of Breaking Load in Raw Cottons

Sample no.	Sample Particulars	Gauge length mm	Constants of calculated frequency curve: m	n	Range a	Goodness of fit chi-Square (x^2)	Mean Single Fibre Breaking Load, g	Standard deviation, g	% Coefficient of Variation of breaking load, g	Mode, g	Skewness
		5	3.5	4.2	21.7	7.6	10.0	3.7	36.9	9.5	0.14
1	Cotton 888	10	3.0	4.3	19.5	4.2	8.0	3.4	42.2	7.4	0.18
	(p_M = 96)	20	3.3	6.1	20.4	15.5	6.0	2.5	42.5	6.3	0.12
2	Egyptian	2	5.3	14.8	25.2	14.1	6.7	2.4	40.0	6.0	0.29
	Karnak	6	2.9	3.5	11.5	8.1	5.2	2.1	40.0	5.0	0.09
	(p_M = 90)	10	3.3	5.2	12.8	6.8	5.0	2.0	40.8	4.5	0.25
		5	2.6	6.0	14.6	10.1	4.4	2.2	49.4	3.5	0.40
3	Cotton 193	10	3.0	14.6	24.5	5.3	3.8	2.1	55.8	3.1	0.33
	(p_M = 73)	15	3.3	16.4	17.5	18.4	3.1	1.6	51.8	2.3	0.50

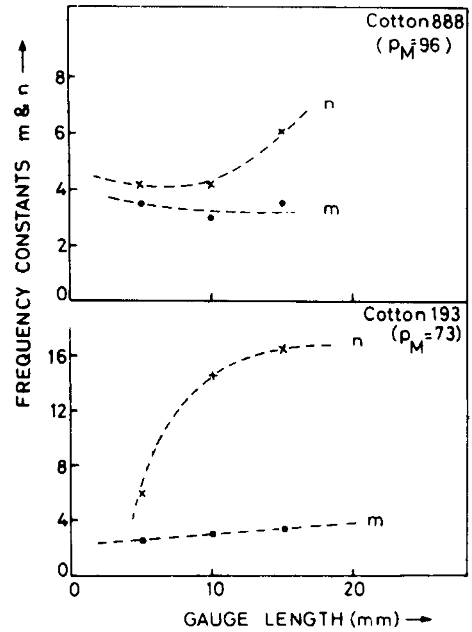

Fig. 5. Plot of m and n vs. gauge length for three raw cottons.

curves have a strong, positive skew, with the exception of the highly mature 875 cotton, which shows a symmetric distribution. In fact, the skewness shows a direct variation with fiber maturity (Table III). The chi-square values obtained in Table III suggest that, in most cases, β distributions fit the observed frequency curves very accurately.

The effect of test gauge length on the variability of breaking load in three of the raw cottons is shown in Table IV and illustrated in Figure 4. It can be seen that the mean, mode, and standard deviation of the breaking load decrease with increasing gauge length. The C.V. of the breaking load shows a trend to increase with increasing gauge length, but the skewness does not show any consistent variation with gauge length. In general, however, the frequency curves for immature cotton show greater asymmetry than the corresponding curves for mature cotton, at the corresponding gauge length. The chi-square values obtained in Table IV again confirm the accuracy of fit of β functions to the observed breaking load histograms of raw cottons, obtained using different gauge lengths.

Figure 5 shows a plot of m and n against gauge length for two raw cottons. At different gauge lengths used, n is always greater than m in all cases, indicating that all the frequency curves in Figure 4 are characterized by a positive skew. The difference between n and m is larger for medium-maturity cotton (193) than for a highly mature cotton (888). The conclusion that the asymmetry of breaking load frequency curves increases with decreasing fiber maturity of cotton is now confirmed at different gauge lengths also (Fig. 5 and Table IV).

DISCUSSION

Breaking load distributions have been obtained for several raw cottons at different gauge lengths. Though many of these distributions have a strong, positive skew, they are not multipeaked. These distributions are found to be of the β type (same as Pearson Type I) on curve fitting. A description of statistical terms used in curve fitting is warranted here. The parameters m and n determine the overall shape and the difference between m and n, the skewness of the frequency curves. The range a is simply the difference between the greatest and the least values of the variable, i.e., breaking load. It is not suitable as an estimate of variability of breaking load because it depends solely on the extreme values. The standard deviation, on the other hand, is a much better measure of dispersion of the breaking load because it is based on all the observations. The C.V. is the standard deviation of the distribution expressed as a percentage of the mean. The mode is the most frequently occurring value of the breaking load in the distribution, i.e., the breaking load at which the peak of a smooth curve drawn through the histogram occurs. Skewness gives a measure of the lopsidedness of the distribution. β_1, β_2, and k are certain functions of moments actually used for classifying the frequency curves. The square root of β_1 can also be used as a measure of skewness. Kurtosis is equal to ($\beta_2 - 3$) and can give an estimate of the peakedness and breadth of the frequency curve.

If the cotton hair were perfect, the rupture of each would involve (a) the elastic properties of cellulose and (b) the dimensions of the cross section of the fibers. In practice, cotton fiber rupture is strongly dependent upon the number of weak points along the length of the fiber. In fact, the initial portion of the breaking load frequency curve (i.e., the portion towards low breaking loads) is influenced mostly by the weak points and structural abnormalities of cotton fibers such as voids, flaws, stress concentrations within the fiber, cracks, portions damaged by fungi or by ginning of fibers, etc. In the present work, no attempt has been made to measure the wall thickness at the actual point of rupture.

The rupture strength of cotton hair never approaches the theoretical strength, i.e., the rupture strength of its chemical bonds, and depends on the length or volume of the sample tested. Cotton fiber is so heterogeneous and structurally complex that it is extremely tedious to consider it to be composed of elements which can be arranged to develop breaking strength models. In other words, from the observed and calculated distributions of the breaking load, it is nearly impossible to determine the distribution of flaws or weak points along the length of the fiber.

The decrease of fiber strength with increase in gauge length is known as the *weak link effect,* which was introduced by Peirce[12] as follows: "the strength of a chain is that of its weakest link." The presence of a distribution of weak points of varying magnitudes accounts for the experimentally established decrease in fiber strength with increasing test length, since the probability of encountering a fatal weak point increases as the length increases. A fiber can be modelled solely as a system of N elements or links. If the distributions of elemental strengths is assumed to be uniform, a *parallel model* of element configuration leads to a β distribution of fiber breaking stresses. Filaments of viscose, cotton, and carbon cannot be represented as systems of simple links connected in series, and it has been shown[13] that the weak link theory is an inadequate model for these.

CONCLUSIONS AND SUMMARY

The β distribution gives an excellent fit to the observed breaking load frequency curves of several raw cottons. The breaking load frequency curves of raw cottons are asymmetric with a positive skew. The skewness decreases with increasing maturity until the breaking load distribution becomes symmetric for almost 100% mature cotton (875). In fact, as the maturity of raw cotton is increased, the range a becomes smaller and smaller, the mode gradually shifts to higher load values, and the distributions become more and more symmetric (at any given gauge length). With increasing fiber maturity of cotton, an increase in mean breaking load and a decrease in C.V. of the breaking load is observed.

Of the frequency constants m and n, m is practically independent of maturity, work of rupture, and breaking tenacity, whereas n bears a strong relationship with each of these fiber characteristics. In general, n is always found to be greater than m, and the difference between these two parameters becomes larger as the maturity of the cotton is decreased. For various raw cottons, the mean, mode, and standard deviation of the breaking load distribution decrease with a progressive increase in gauge length.

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